Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 2

15 May 2026

Zone A morning | **Zone B** morning | **Zone C** morning

2 hours

Instructions to students

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

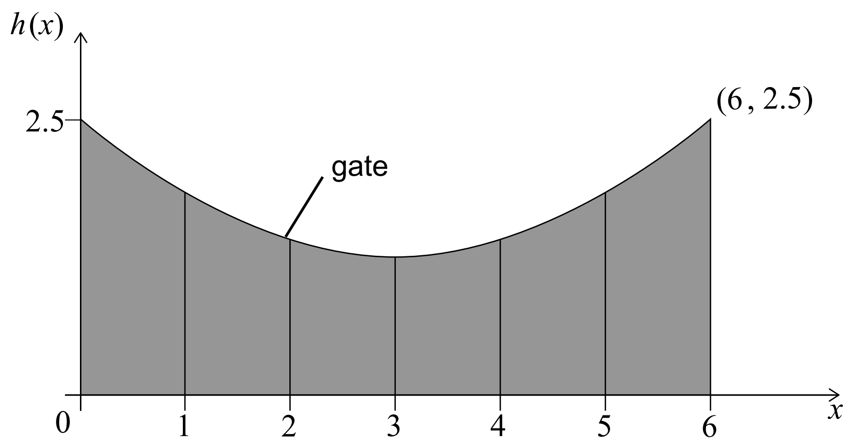
Aisha wants to paint the gate to her house. She only wants to paint the gate on one side.

The gate is $6\,\mathrm{m}$ wide and its height at each side is $2.5\,\mathrm{m}$. The top edge of the gate is in the shape of a parabola and the height, h metres, of the gate is modelled by the equation

$$h(x) = \frac{1}{9}x^2 - \frac{2}{3}x + 2.5, \text{ for } 0 < x < 6,$$

where x is the distance, in metres, from the left side of the gate. This is shown in the following diagram.

diagram not to scale



(a) Find the minimum height of the gate. [2]

$h(x)$ is increasing for $a < x < b$.

(b) Find the value of a and of b. [1]

(This question continues on the following page)

(Question 1 continued)

Aisha uses the trapezoidal rule to find the approximate area of the gate.

She uses $h(x)$ to find the height at 1 m intervals, and records the results in the following table.

$\boldsymbol{x}$ **(m)**	0	1	2	3	4	5	6
$\boldsymbol{h}$ **(m)**	2.5	$\frac{35}{18}$	$\frac{29}{18}$	p	$\frac{29}{18}$	q	2.5

(c) (i) Write down the value of p and of q.

(ii) Calculate the approximate area of the gate using the trapezoidal rule. [5]

Aisha realizes that this approximate area is an overestimate of the area of the gate.

(d) Without calculation, explain why the trapezoidal rule will produce an overestimate in this case. [1]

Aisha then calculates the exact area, A, of the gate, according to her model, $h(x)$.

(e) (i) Write down an integral expression for A.

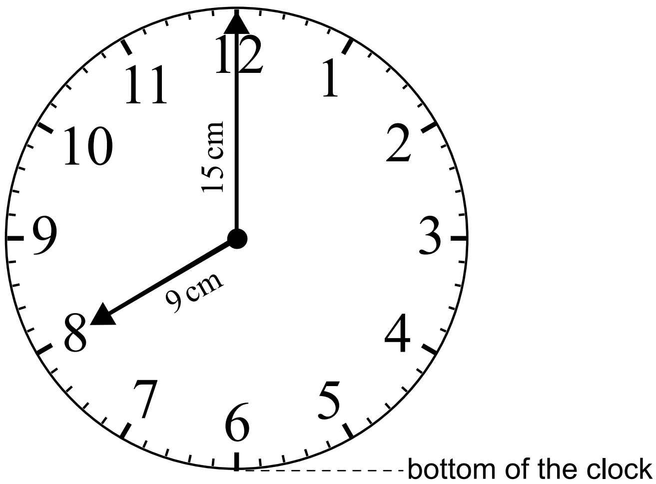
(ii) Calculate the value of A. [4]

(f) Find the absolute percentage error when using the value from part (c)(ii) as an approximation for A. [2]

2. [Maximum mark: 14]

The circular face of a wall clock has a diameter of $30\,\text{cm}$. The minute hand of the clock is $15\,\text{cm}$ long and the hour hand is $9\,\text{cm}$ long. The clock is initially set to $8{:}00$. This is shown in the following diagram.

diagram not to scale



(a) Calculate the distance between the endpoints of the two hands of the clock at $8{:}00$. [4]

Both hands of the clock rotate at a constant rate as time passes, to indicate the time at a particular moment. The minute hand makes 1 rotation in 1 hour, while the hour hand makes 1 rotation in 12 hours.

As the minute hand moves, the height of its endpoint, $h(t)$, above the bottom of the clock, can be modelled as the cosine function

$$h(t) = a\cos\left((bt)^\circ\right) + d \text{ where } a,\ b,\ d > 0,$$

where t is the time after $8{:}00$, measured in minutes, and bt is measured in degrees.

(b) Find the value of

(i) a.

(ii) b.

(iii) d. [4]

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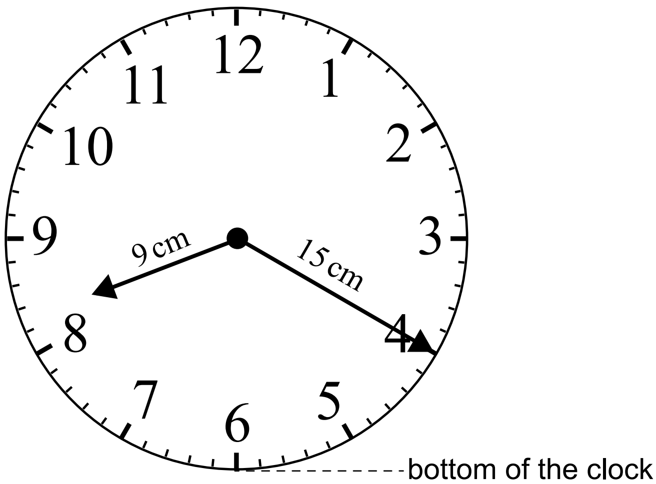
(Question 2 continued)

Later, the clock shows the time $8{:}20$.

diagram not to scale



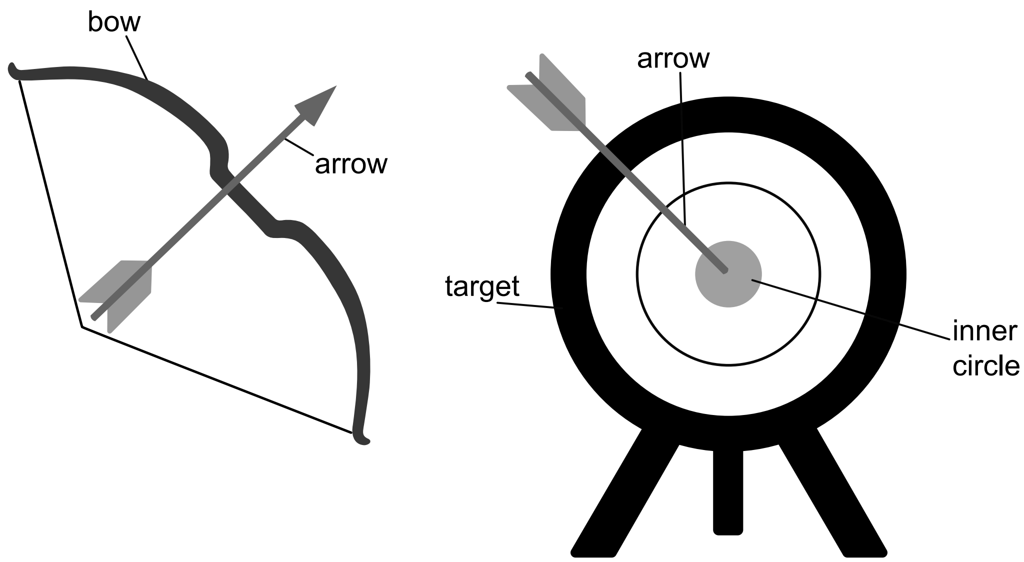
(c) Use the model, or otherwise, to calculate the height of the endpoint of the minute hand above the bottom of the clock at $8{:}20$. [2]

(d) Explain why the distance between the endpoints of the two hands of the clock is greater at $8{:}20$ than at $8{:}00$. [1]

(e) Calculate the difference in height between the endpoints of the two hands of the clock at $8{:}20$. [3]

3. [Maximum mark: 17]

Mores joins an archery club, where he shoots arrows with his bow towards a target.



The probability that he hits the target with his first arrow is 0.34.

If Mores hits the target with his first arrow, the probability that he hits the target with his second arrow is 0.72.

If Mores does **not** hit the target with his first arrow, the probability that he hits the target with his second arrow is 0.68.

(a) Draw and complete a tree diagram to represent all possible outcomes from Mores shooting two arrows. [3]

(b) Find the probability that Mores hits the target **at most once** after shooting two arrows. [3]

(c) Find the probability that Mores hits the target with his second arrow. [2]

(This question continues on the following page)

(Question 3 continued)

Dipali is an expert archer. The probability that his arrow hits the inner circle of the target on any shot is 0.94. The arrow hitting the inner circle on one shot is independent of the arrow hitting the inner circle on any other shot.

Dipali takes 10 shots at the target.

(d) Find the probability that he hits the inner circle exactly 9 times. [2]

(e) Find the probability that he hits the inner circle at least 7 times. [2]

In a tournament, Dipali shoots a total of 100 arrows at the target.

(f) Find the expected mean and standard deviation of the number of times Dipali hits the inner circle during the tournament. [3]

The probability that Dipali hits the inner circle of the target with at most k arrows is greater than 0.25.

(g) Find the smallest possible value of k. [2]

4. [Maximum mark: 14]

A particle moves along a two-dimensional plane relative to an origin, O. Its position vector at time t is

$$\boldsymbol{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4\sin(2t) + 3\cos(2t) \\ 4\cos(2t) - 3\sin(2t) \end{pmatrix}.$$

(a) Write down the coordinates of the particle's initial position. [2]

(b) Show that the particle remains at a fixed distance from the origin, O, throughout its motion. Calculate this fixed distance. [5]

(c) Find an expression for $\boldsymbol{v}$, the velocity vector of the particle. [3]

(d) Show, by considering the scalar product, that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{r}$ throughout the motion of the particle. [4]

5. [Maximum mark: 17]

The flight of an eagle, swooping vertically downwards to catch a fish in the sea, is modelled by the second order differential equation

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0 \text{, for } 0 \le t \le 10 \text{,}$$

where x, measured in metres, is its vertical displacement above a point, O, at sea level at time t, measured in seconds.

(a) Use the substitution $y = \frac{dx}{dt}$ to write this differential equation as a system of coupled first order differential equations. Write your answer in the form

$$\frac{dx}{dt} = y$$
$$\frac{dy}{dt} = ax + by \text{.}$$ [2]

The coupled differential equations, found in part (a), can be expressed as a matrix equation of the form

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{pmatrix} = \boldsymbol{A}\begin{pmatrix} x \\ y \end{pmatrix}.$$

(b) Write down the 2×2 matrix $\boldsymbol{A}$. [2]

(c) Find the eigenvalues of $\boldsymbol{A}$. [3]

(d) Find the eigenvectors of $\boldsymbol{A}$. [3]

(e) Hence, or otherwise, write down the general solution to the second order differential equation used to model the eagle's swoop. [1]

The eagle starts its vertical swoop with an initial velocity of $-50\,\mathrm{ms}^{-1}$ from a vertical displacement of $20\,\mathrm{m}$ above O.

(f) Show that the eagle's vertical displacement from O is given by

$$x(t) = 30e^{-2t} - 10e^{-t}.$$ [3]

(g) (i) Find the time at which the eagle enters the water.

(ii) Find the minimum vertical displacement of the eagle. [3]

6. [Maximum mark: 17]

Allison works at a call centre. The number of incoming calls she has in a 10-minute period follows a Poisson distribution with mean 1.25.

(a) Find the probability that Allison has more than 2 incoming calls in a 10-minute period. [2]

Allison is allowed a 20-minute break at mid-morning.

(b) Find the probability that Allison has no incoming calls during this break time. [3]

Allison is training her colleague, Jean. The number of incoming calls Jean has in a 10-minute period follows a Poisson distribution with mean 0.82, and is independent of the number of calls Allison has.

(c) Find the probability that Allison and Jean together have fewer than 4 calls in a randomly chosen 30-minute period. [3]

Once Allison has answered a call, she is allowed a 10-minute period in which to complete the call.

The times Allison takes to complete a call have been found to follow a normal distribution with mean 12 minutes and standard deviation 3 minutes. The times are assumed to be independent of each other.

(d) Calculate the probability that Allison does not complete a call within her allowed time period. [2]

As part of her performance review, 6 of Allison's calls are selected at random. The times she took to resolve these 6 calls are logged and the mean time found.

(e) Calculate the probability that this mean is less than 10 minutes. [4]

The times Jean, Allison's trainee, takes to resolve a call have been found to follow a normal distribution with mean 11 minutes and standard deviation 2 minutes. The times are assumed to be independent of each other and of Allison's times.

(f) Find the probability that Allison takes longer than Jean to resolve a randomly selected call. [3]

7. [Maximum mark: 16]

The expression $\frac{1}{x(2-x)}$ can be written as $\frac{A}{x}+\frac{A}{(2-x)}$.

(a) Show that $A=\frac{1}{2}$. [2]

The given expression and the answer to part (a) can be used to find the general solution to the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t}=x(2-x).$$

(b) Use separation of variables to find the general solution to this differential equation. [6]

(c) Find the solution to the differential equation that satisfies the initial condition that $x = 1$ at $t=0$. Give your answer in the form $t=f(x)$. [2]

Consider the case where $0<x<2$.

(d) State the reason why modulus signs are no longer required in the $f(x)$ you found in part (c). [1]

(e) By first removing the modulus signs from $f(x)$, find x in terms of t. [5]